Physics 11: Additional Equations for the Final Celebration

Chapter 10: Energy and Work

Work: $W = Fd\cos\theta$

- ⇒ The unit of work is the Joule (J). 1 J = 1 Nm = 1 kg m^2/s^2
- ⇒ Work can be +, -, or 0.

Work Energy Theorem: The total energy of a system changes by the amount of work done on it:

$$\Delta E = \Delta K + \Delta U_g + \Delta U_s + \Delta E_{th} + \Delta E_{chem} + \ldots = W$$

Law of conservation of energy: The total energy of an isolated system (no work is done on the system) remains constant:

$$\Delta E = \Delta K + \Delta U_g + \Delta U_s + \Delta E_{th} + \Delta E_{chem} + \ldots = 0$$

Kinetic Energy: $K = {}^1\!/_2 mv^2$

Gravitational Potential Energy: $U_g = mgy$

Elastic Potential Energy: $U_s = \frac{1}{2}kx^2$

Thermal energy: $\Delta E_{th} = f_k \Delta x$

Conservation of Mechanical Energy: The mechanical energy of an isolated system without friction is conserved:

$$K_f + (U_g)_f + (U_s)_f = K_i + (U_g)_i + (U_s)_i$$

⇒ If there is friction, the total energy is conserved:

$$K_f + (U_g)_f + (U_s)_f + \Delta E_{th} = K_i + (U_g)_i + (U_s)_i$$

Power: The rate at which energy is transformed or work is done:

$$P = {}^{\Delta E}\!/_{\Delta t}$$

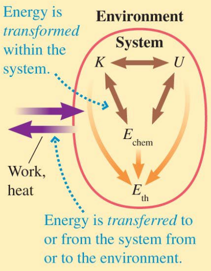
$$P = {}^{W}\!/_{\Delta t}$$

Basic Energy Model

Within a system, energy can be **transformed** between various forms.

Energy can be **transferred** into or out of a system in two basic ways:

- **Work:** The transfer of energy by mechanical forces.
- **Heat:** The nonmechanical transfer of energy from a hotter to a colder object.



Energy is *transformed* within the system.

Energy is *transferred* to or from the system from or to the environment.

Conservation of Energy

When work W is done on a system, the system's total energy changes by the amount of work done. In mathematical form, this is the **work-energy equation:**

$$\Delta E = \Delta K + \Delta U_g + \Delta U_s + \Delta E_{th} + \Delta E_{chem} + \cdots = W$$

A system is **isolated** when no energy is transferred into or out of the system. This means the work is zero, giving the **law of conservation of energy:**

$$\Delta K + \Delta U_g + \Delta U_s + \Delta E_{th} + \Delta E_{chem} + \cdots = 0$$

Solving Energy Conservation Problems

PREPARE Choose your system so that it's isolated. Draw a before-and-after visual overview.

SOLVE

- If the system is isolated and there's no friction, then mechanical energy is conserved:

$$K_f + (U_g)_f + (U_s)_f = K_i + (U_g)_i + (U_s)_i$$

- If the system is isolated but there's friction present, then the total energy is conserved:

$$K_f + (U_g)_f + (U_s)_f + \Delta E_{th} = K_i + (U_g)_i + (U_s)_i$$

ASSESS Kinetic energy is always positive, as is the change in thermal energy.

Kinetic energy is an energy of motion:

$$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Translational

Rotational

Potential energy is energy stored in a system of interacting objects.

- **Gravitational potential energy:** $U_g = mgy$
- **Elastic potential energy:** $U_s = \frac{1}{2}kx^2$

Mechanical energy is the sum of a system's kinetic and potential energies:

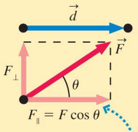
$$\text{Mechanical energy} = K + U = K + U_g + U_s$$

Thermal energy is the sum of the microscopic kinetic and potential energies of all the molecules in an object. The hotter an object, the more thermal energy it has. When kinetic (sliding) friction is present, the increase in the thermal energy is $\Delta E_{th} = f_k \Delta x$.

Work is the process by which energy is transferred to or from a system by the application of mechanical forces.

If a particle moves through a displacement $\vec{d}$ while acted upon by a constant force $\vec{F}$, the force does work

$$W = F_{\parallel}d = Fd\cos\theta$$



Only the component of the force parallel to the displacement does work.

Power is the rate at which energy is transformed . . .

$$P = \frac{\Delta E}{\Delta t}$$

Amount of energy transformed

Time required to transform it

. . . or at which work is done.

$$P = \frac{W}{\Delta t}$$

Amount of work done

Time required to do work

Chapter 13: Fluids

Density: $\rho = {}^{m}\!/_{V}$ $\rho_{water} = 1.000 \times 10^3\ {}^{kg}\!/_{m^3}$

$m = \rho V$

Pressure:

Pressure: $p = \dfrac{F}{A}$ $p_{atm} = 1.013 \times 10^5$ Pa = 1 atm

Pressure in a Static Fluid: $p = p_0 + \rho g d$

Pascal's Principle: If the pressure at one point in an incomprehensible fluid is changed, the pressure at every other point in the fluid changes by the same amount.

Buoyancy:

Archimede's Principle: the magnitude of the buoyant force on an object partially or completely immersed in a fluid equals the weight of the fluid displaced by the object

Buoyant Force: $F_B = \rho_{fluid} V_{sub} g$

⇒ if an object is completely submerged, $V_{sub} = V_{obj}$

⇒ if an object is floating, $F_B = w = mg$

Fluids in Motion:

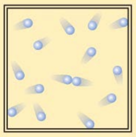
Equation of Continuity: $A_1 v_1 = A_2 v_2$

Bernoulli's Equation: $p_1 + {}^{1}\!/_{2}\,\rho v_1^2 + \rho g y_1 = p_2 + {}^{1}\!/_{2}\,\rho v_2^2 + \rho g y_2$

Bernoulli's Principle: where the speed of a fluid increases, the pressure in the fluid decreases

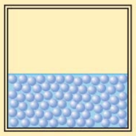
Fluid Statics

Gases



- Freely moving particles
- Compressible
- Pressure mainly due to particle collisions with walls

Liquids



- Loosely bound particles
- Incompressible
- Pressure due to the weight of the liquid
- Hydrostatic pressure at depth d is $p = p_0 + \rho g d$
- The pressure is the same at all points on a horizontal line through a liquid (of one kind) in hydrostatic equilibrium

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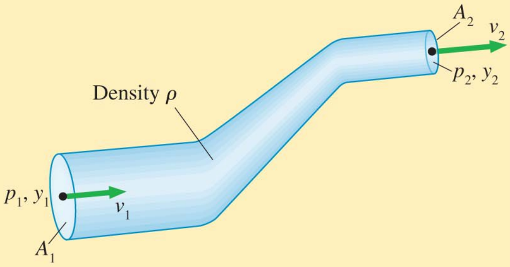
Fluid Dynamics



Ideal-fluid model

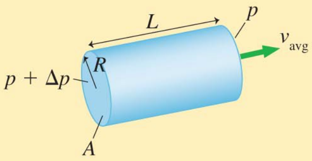
- Incompressible
- Smooth, laminar flow
- Nonviscous

Equation of continuity

Volume flow rate $Q = \dfrac{\Delta V}{\Delta t} = v_1 A_1 = v_2 A_2$

Bernoulli's equation is a statement of energy conservation:

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$



Poiseuille's equation governs viscous flow through a tube:

$$Q = v_{\text{avg}} A = \frac{\pi R^4 \Delta p}{8 \eta L}$$

Density $\rho = m/V$, where m is mass and V is volume.

Pressure $p = F/A$, where F is force magnitude and A is the area on which the force acts.

- Pressure exists at all points in a fluid.
- Pressure pushes equally in all directions.
- Gauge pressure $p_g = p - 1$ atm.

Viscosity η is the property of a fluid that makes it resist flowing.

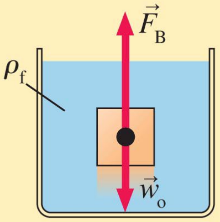
Buoyancy is the upward force of a fluid on an object immersed in the fluid.

Archimedes' principle: The magnitude of the buoyant force equals the weight of the fluid displaced by the object.

Sink: $\rho_{\text{avg}} > \rho_f \quad F_B < w_o$

Float: $\rho_{\text{avg}} < \rho_f \quad F_B > w_o$

Neutrally buoyant: $\rho_{\text{avg}} = \rho_f \quad F_B = w_o$



Chapter 16: Waves and Sound

Waves:

Transverse Wave: the disturbance occurs perpendicular to the direction of travel of the wave

Longitudinal Wave: the disturbance occurs parallel to the direction of travel of the wave

$$f = \frac{1}{T} \qquad T = \frac{1}{f} \qquad v = \frac{\lambda}{T} \qquad v = \lambda f$$

⇒ if the frequency is increased, the wavelength is decreased but wave speed doesn't change

Speed of waves on a string: $v = \sqrt{\frac{T_s}{\mu}} \qquad \mu = \frac{m}{L}$

Doppler Effect: $f_0 = f_s \left(\frac{1 \pm {}^{v_0}\!/_{v}}{1 \mp {}^{v_s}\!/_{v}} \right)$

Chapter 17: Linear Superposition and Interference

Linear Superposition:

The Principle of Linear Superposition: when two or more waves are present at the same place at the same time, the resultant disturbance is the sum of the disturbances from the individual waves

⇒ for 2 wave sources vibrating in phase:

$\Delta L = n\lambda \qquad n = 0, 1, 2, \ldots$ constructive interference

$\Delta L = (n + {}^{1}\!/_{2})\lambda \qquad n = 0, 1, 2, \ldots$ destructive interference

Beats: $f_{beats} = |f_1 - f_2|$

Standing Waves:

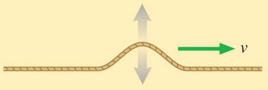
⇒ for standing waves on a string fixed at both ends:

$$\lambda_m = \frac{2L}{m} \qquad f_m = m\left(\frac{v}{2L}\right) \qquad m = 1, 2, 3, \ldots$$

The Wave Model

This model is based on the idea of a **traveling wave,** which is an organized disturbance traveling at a well-defined **wave speed** v.

- In **transverse waves** the particles of the medium move *perpendicular* to the direction in which the wave travels.



- In **longitudinal waves** the particles of the medium move *parallel* to the direction in which the wave travels.

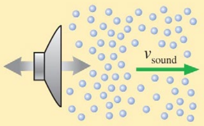
A wave transfers energy, but there is no material or substance transferred.

Mechanical waves require a material **medium.** The speed of the wave is a property of the medium, not the wave. The speed does not depend on the size or shape of the wave.

- For a **wave on a string,** the string is the medium.

$$v_{\text{string}} = \sqrt{\frac{T_s}{\mu}}$$

- A **sound wave** is a wave of compressions and rarefactions of a medium such as air.



In a gas:

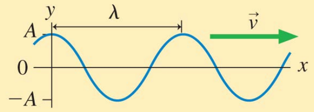
$$v_{\text{sound}} = \sqrt{\frac{\gamma RT}{M}}$$

Electromagnetic waves are waves of the electromagnetic field. They do not require a medium. All electromagnetic waves travel at the same speed in a vacuum, $c = 3.00 \times 10^8$ m/s.

Graphical representation of waves

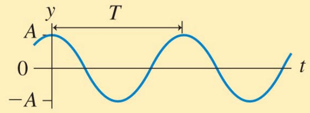
A **snapshot graph** is a picture of a wave at one instant in time. For a periodic wave, the **wavelength** λ is the distance between crests.

Fixed t:



A **history graph** is a graph of the displacement of one point in a medium versus time. For a periodic wave, the **period** T is the time between crests.

Fixed x:

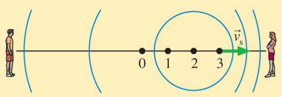


The **Doppler effect** is a shift in frequency when there is relative motion of a wave source (frequency f_0, wave speed v) and an observer.

Moving source, stationary observer:

Receding source:

$$f_- = \frac{f_0}{1 + v_s/v}$$



Approaching source:

$$f_+ = \frac{f_0}{1 - v_s/v}$$

Moving observer, stationary source:

Approaching the source: $f_+ = \left(1 + \frac{v_o}{v}\right) f_0$

Moving away from the source: $f_- = \left(1 - \frac{v_o}{v}\right) f_0$

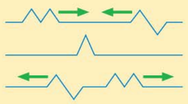
Reflection from a moving object:

For $v_o \ll v$, $\Delta f = \pm 2 f_0 \frac{v_o}{v}$

When an object moves faster than the wave speed in a medium, a **shock wave** is formed.

Principle of Superposition

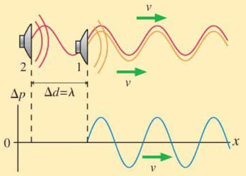
The displacement of a medium when more than one wave is present is the sum of the displacements due to each individual wave.



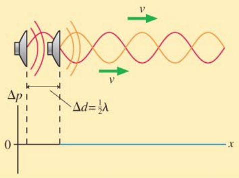
Interference

In general, the superposition of two or more waves into a single wave is called interference.

Constructive interference occurs when crests are aligned with crests and troughs with troughs. We say the waves are in phase. It occurs when the path-length difference Δd is a whole number of wavelengths.

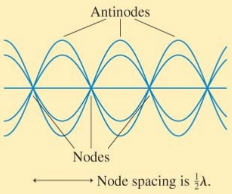


Destructive interference occurs when crests are aligned with troughs. We say the waves are out of phase. It occurs when the path-length difference Δd is a whole number of wavelengths plus half a wavelength.



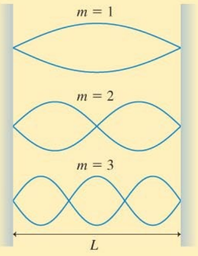
Standing Waves

Two identical traveling waves moving in opposite directions create a standing wave.



The boundary conditions determine which standing-wave frequencies and wavelengths are allowed. The allowed standing waves are **modes** of the system.

A **standing wave on a string** has a node at each end. Possible modes:



$$\lambda_m = \frac{2L}{m} \qquad f_m = m\left(\frac{v}{2L}\right) = mf_1$$

$$m = 1, 2, 3, \ldots$$

Chapter 18: Ray Optics

The Reflection of Light:

⇒ in optics, all angles are measure with respect to the normal

Law of Reflection: $\theta_r = \theta_i$

Images from Plane Mirrors:

⇒ the image created by a plane mirror is virtual, upright, the same size as the object, and as far behind the mirror as the object is in front of it

Images from Spherical Mirrors:

⇒ a convex mirror always produces a virtual, reduced, and upright image

⇒ a concave mirror can produce a:

real, enlarged, inverted image (if object is between C and F)
real, reduced, inverted image (if object is beyond C)
virtual, enlarged, upright image (if object is between F and mirror)

⇒ from a single mirror, real images are always inverted and virtual images are always upright

Mirror and Magnification Equations:

mirror equation: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ magnification equation: $m = \frac{h'}{h} = -\frac{s'}{s}$

$f > 0$	concave mirror
$f < 0$	convex mirror

$s > 0$	real object (in front of mirror)	$s' > 0$	real image (in front of mirror)
$s < 0$	virtual object (behind mirror)	$s' < 0$	virtual image (behind mirror)

$m > 0$	image is upright	$\lvert m \rvert > 1$	image is enlarged
$m < 0$	image is inverted	$\lvert m \rvert < 1$	image is reduced

The Refraction of Light:

Index of Refraction: $n = \dfrac{\text{speed of light in vacuum}}{\text{speed of light in the material}} = \dfrac{c}{v}$

Snell's Law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

Total Internal Reflection:

⇒ total internal reflection can only occur if $n_2 < n_1$

Critical Angle: $\theta_c = \sin^{-1}\left({}^{n_2}\!/\!{}_{n_1} \right)$

Images from Lenses:

⇒ a diverging lens always produces a virtual, reduced, and upright image

⇒ a converging lens can produce a:

real, enlarged, inverted image (if object is between F and 2F)
real, reduced, inverted image (if object is beyond 2F)
virtual, enlarged, upright image (if object is between F and lens)

⇒ from a single lens, real images are always inverted and virtual images are always upright

Thin Lens and Magnification Equations:

thin lens equation: $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$ magnification equation: $m = \dfrac{h'}{h} = -\dfrac{s'}{s}$

$f > 0$	converging lens
$f < 0$	diverging lens

$s > 0$	real object (on the left of the lens)	$s' > 0$	real image (on the right of the lens)
$s < 0$	virtual object (on the right of the lens)	$s' < 0$	virtual image (on the left of the lens)

$m > 0$	image is upright	$\lvert m \rvert > 1$	image is enlarged
$m < 0$	image is inverted	$\lvert m \rvert < 1$	image is reduced

Chapter 20: Electric Fields and Forces

charge on a proton: $q_p = +e = 1.60 \times 10^{-19}$ C
charge on an electron: $q_e = -e = -1.60 \times 10^{-19}$ C

charge is quantized: $q = \pm ne$ n = 0, 1, 2, …

Charged Objects:

Law of Conservation of Electric Charge: during any process, the net electrical charge of an isolated system remains constant

⇒ like charges repel and unlike charges attract each other

⇒ there are three ways to charge an object: charging by friction, charging by induction, and charging by contact

Coulomb's Law:

$$F_{1on2} = F_{2on1} = \frac{k|q_1||q_2|}{r^2} \qquad k = 8.99 \times 10^9 \; {Nm^2}/{C^2}$$

⇒ the direction of the force is along a line connecting the two charges

⇒ if there are more than two charges, the net force on a charge is the vector sum of all the forces on the charge

The Electric Field:

⇒ the electric field is defined as: $\vec{E}\,at\,(x,y,z) = \dfrac{\vec{F}_{onq}\,at\,(x,y,z)}{q}$ where q is a small + test charge

⇒ the force on a charge in an electric field is given by: $\vec{F} = q\vec{E}$

⇒ the electric field from a point charge is given by: $E = \dfrac{k|q|}{r^2}$

$\vec{E}$ points away from a positive charge

$\vec{E}$ points toward a negative charge

Electric Field Lines:

⇒ electric field lines have the following properties:

Electric field lines point in the direction of the electric field

The closer together the field lines, the string the electric field.

Electric field lines point away from a positive charge and towards a negative charge.

Electric field lines never cross.

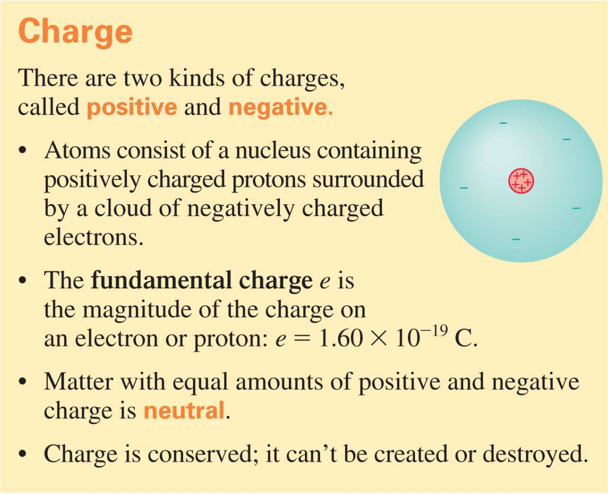
Conductors in Electrostatic Equilibrium:

⇒ A conductor in electrostatic equilibrium (charges at rest) have the following properties:

The electric field inside the conductor is zero.

Any excess charge is located on the surface of the conductor.

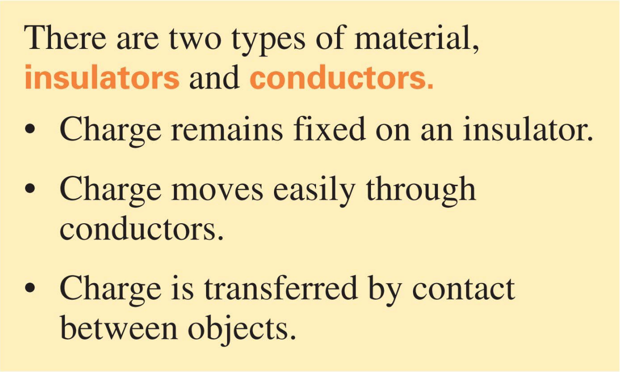
The electric field is perpendicular to the surface.



Charge

There are two kinds of charges, called **positive** and **negative.**

- Atoms consist of a nucleus containing positively charged protons surrounded by a cloud of negatively charged electrons.
- The **fundamental charge** e is the magnitude of the charge on an electron or proton: $e = 1.60 \times 10^{-19}$ C.
- Matter with equal amounts of positive and negative charge is **neutral**.
- Charge is conserved; it can't be created or destroyed.



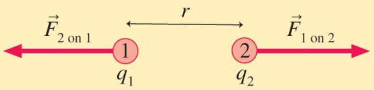
There are two types of material, **insulators** and **conductors.**

- Charge remains fixed on an insulator.
- Charge moves easily through conductors.
- Charge is transferred by contact between objects.

Coulomb's Law

The forces between two charged particles q_1 and q_2 separated by distance r are

$$F_{1\,\text{on}\,2} = F_{2\,\text{on}\,1} = \frac{K|q_1||q_2|}{r^2}$$



where $K = 8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2$ is the **electrostatic constant.** These forces are an action/reaction pair directed along the line joining the particles.

- The forces are repulsive for two like charges, attractive for two opposite charges.
- The net force on a charge is the vector sum of the forces from all other charges.
- The unit of charge is the coulomb (C).

The Electric Field

Charges interact with each other via the electric field $\vec{E}$.

- Charge A alters the space around it by creating an electric field.

- The field is the agent that exerts a force on charge B.

$$\vec{F}_{\text{on B}} = q_{\text{B}}\vec{E}$$

- An electric field is identified and measured in terms of the force on a probe charge q. The unit of the electric field is N/C.

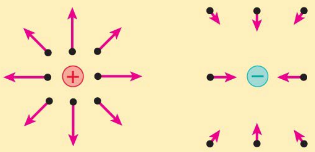
$$\vec{E} = \frac{\vec{F}_{\text{on}\,q}}{q}$$

- The electric field is a vector. The electric field from multiple charges is the vector sum of the fields from the individual charges.

$$\vec{E}_{\text{total}} = \vec{E}_1 + \vec{E}_2 + \cdots$$

Visualizing the electric field

The electric field exists at all points in space.



- An electric field vector shows the field only at one point, the point at the tail of the vector.
- A **field diagram** shows field vectors at several points.
- **Electric field lines:**
  - are always parallel to the field vectors.
  - are close where the field is strong, far apart where the field is weak.
  - go from positive to negative charges.